

# Incompleteness, Temporal Axiom Systems, and the Emergence of Creativity in Artificial Intelligence

Hirofumi Inomata  
inomata@acm.org

## 1 Introduction

This paper investigates the structural relationships between logical systems, Gödelian incompleteness, time-evolving axiom systems, and their implications for artificial intelligence and creativity.

We develop a unified framework in which:

- Logical systems are inherently incomplete,
- Axiom systems may evolve over time,
- Logical futures branch infinitely,
- Artificial intelligence operates over multiple implicit logical systems,
- Creativity emerges from structured selection within incomplete systems.

## 2 Formal Systems and Incompleteness

**Definition 1.** A formal system is a pair  $(\Gamma, R)$  consisting of a set of axioms  $\Gamma$  and inference rules  $R$ .

**Definition 2.** The set of all theorems derivable from  $\Gamma$  is denoted by

$$\text{Th}(\Gamma).$$

**Theorem 1** (Gödel's Incompleteness). Any sufficiently expressive, consistent, effectively axiomatizable formal system is incomplete; that is, there exists a true statement that is not provable within the system.

### 3 Time-Evolving Axiom Systems

**Definition 3.** A time-evolving axiom system is a family  $\{A_t\}_{t \in T}$  such that

$$s \leq t \Rightarrow A_s \subseteq A_t.$$

**Theorem 2.** If infinitely many independent axioms can be added after a time  $t_0$ , then the future logical systems generated from  $A_{t_0}$  are infinite in number.

*Proof.* Let  $\beta_0, \beta_1, \beta_2, \dots$  be independent sentences. Define

$$A_t^{(n)} = \begin{cases} A_{t_0} & t \leq t_0 \\ A_{t_0} \cup \{\beta_m \mid m \geq n, \tau_m \leq t\} & t > t_0 \end{cases}$$

Then distinct  $n$  produce distinct theorem sets, hence infinitely many futures.  $\square$

### 4 Order Dependence of Axiom Acquisition

**Theorem 3.** If axioms  $\alpha$  and  $\beta$  are independent over  $G$ , then different acquisition orders yield different intermediate logical systems.

*Proof.* Consider  $G \cup \{\alpha\}$  and  $G \cup \{\beta\}$ . Since  $\alpha$  is not derivable from  $G \cup \{\beta\}$ , their theorem sets differ.  $\square$

### 5 Artificial Intelligence as a Logical System

AI systems are not fixed formal systems but can be viewed as:

- possessing implicit axiom-like knowledge,
- applying probabilistic inference rules,
- dynamically updating context over time.

**Proposition 1.** A fixed snapshot of an AI system can be modeled as a formal system and is therefore subject to Gödelian incompleteness.

**Proposition 2.** A dynamically evolving AI system is not a single formal system but a sequence of systems, each individually incomplete.

## 6 Multi-Agent Systems and Incompleteness

**Theorem 4.** A collection of interacting AI systems cannot eliminate Gödelian incompleteness.

*Proof.* The union of all axioms and inference rules forms a larger system, to which Gödel's theorem applies.  $\square$

## 7 Self-Reference and Limits of Knowledge

**Proposition 3.** Any sufficiently expressive self-referential system contains true statements that it cannot prove.

This applies equally to AI systems capable of reasoning about their own reasoning processes.

## 8 Creativity and Incompleteness

**Definition 4.** Let  $\Omega$  be a space of possible outputs and  $V : \Omega \rightarrow \mathbb{R}$  an evaluation function.

**Definition 5.** Creativity is the process of selecting structured outputs from  $\Omega$  according to  $V$ .

**Proposition 4.** In a complete system, creativity collapses, as all outcomes are predetermined.

**Proposition 5.** Incompleteness generates a space of possible extensions, enabling creativity.

## 9 Randomness vs Creativity

**Proposition 6.** Randomness corresponds to unguided sampling from  $\Omega$ , while creativity corresponds to guided selection under  $V$ .

## 10 Evaluation Functions and AI

**Proposition 7.** Current AI systems do not possess fully autonomous evaluation functions but operate under externally defined objectives.

**Proposition 8.** A fully autonomous evaluation function would require a self-originating value system, which leads to infinite regress unless grounded.

## 11 Conclusion

We conclude:

- Incompleteness is a structural property of all sufficiently expressive logical systems.
- Temporal evolution leads to branching futures of logical systems.
- Multiple interacting systems do not eliminate incompleteness.
- Creativity emerges from structured selection within incomplete systems.
- Evaluation functions define the direction of creative processes.

Thus, incompleteness is not merely a limitation but a necessary condition for the existence of open-ended reasoning and creativity.